Natural Sciences 102 Problem Set 6 Solutions

Timothy Donaghy, Alberto Vallinotto

May 23, 2005

Problem 1

a) The H-R diagram can be constructed by measuring two properties of stars, their apparent magnitude and their color, that can always be measured even when the distance is not (yet) known. The H-R diagram is used to find the distance to clusters of stars by identifying its Main Sequence and comparing it with the Main Sequence in clusters for which we already know the distance. Like many of the distance indicators used by astronomers, the Main Sequence is useful because it acts as a "standard candle" – that is, something for which we can discover its intrinsic luminosity.

For example, say we know the distance to one cluster of stars via parallax, and we wish to find the distance to another cluster of the same type. We can assume that in two star clusters, stars at a certain point along the Main Sequence will have the same luminosity. Therefore, if there is a difference in the apparent magnitudes between the two, it must be entirely due to the fact that one cluster is farther away than the other.

b) We see from the H-R diagram that the Big Cunha cluster is on average about 5 magnitudes fainter than M105. First we transform the *difference* in magnitudes into a *ratio* of intensities (or brightnesses).

$$\begin{aligned} m_{cunha} - m_{M105} &= -2.5\log\left(\frac{I_{cunha}}{I_{M105}}\right) \\ 5 &= -2.5\log\left(\frac{I_{cunha}}{I_{M105}}\right) \\ -2 &= \log\left(\frac{I_{cunha}}{I_{M105}}\right) \\ 10^{-2} &= 0.01 = \frac{I_{cunha}}{I_{M105}}, \end{aligned} \tag{1}$$

Now, that we know the ratio of the intensities, we can convert that into a ratio of distances, using our old friend:

$$I = \frac{L}{4\pi D^2}. \tag{2}$$

Thus, our ratio of brightnesses can be written as

$$0.01 = \frac{I_{cunha}}{I_{M105}} = \frac{\frac{L_{cunha}}{4\pi D^2_{cunha}}}{\frac{L_{M105}}{4\pi D^2_{M105}}}. \tag{3}$$

We can then use the fact that the Main Sequence is a standard candle, that is, $L_{cunha} = L_{M105}$, and cancel the L's and the 4π's. This gives us

$$0.01 = \frac{I_{cunha}}{I_{M105}} = \frac{D^2_{M105}}{D^2_{cunha}}, \tag{4}$$

or, taking the square-root of both sides,

$$0.1 = \frac{D_{M105}}{D_{cunha}} = \frac{1000\text{pc}}{D_{cunha}}. \tag{5}$$

Solving then gives us our answer: $D_{cunha} = 10000$ pc.

Problem 2

a) The Erwinium emits with a wavelength of $\lambda_0 = 5000\ \mathring{A}$ in the laboratory while the one measured coming from the Vallinotto galaxy has a wavelength of $\lambda_0 = 5500\ \mathring{A}$. We can then calculate the receding velocity of Vallinotto by

$$\begin{aligned} \frac{v}{c} &= \frac{\lambda - \lambda_0}{\lambda_0} = \frac{5500 - 5000}{5000} = 10^{-1} \\ &\Rightarrow \quad v = \frac{c}{10} = 3 \cdot 10^7 \text{ m/s} = 3 \cdot 10^4 \text{ km/s}. \end{aligned} \tag{6}$$

b) Having the recessional velocity it is then possible to figure out the distance of Vallinotto just by applying Hubble's law

$$v = H_0 d \Rightarrow d = \frac{v}{H_0} = \frac{3 \cdot 10^4 \text{ km s}^{-1}}{50 \text{ km s}^{-1} \text{ Mpc}^{-1}} = 600 \text{ Mpc}. \tag{7}$$

Problem 3

Now, since the wavelength reflected by Andrew's gunrack is measured by the sheriff's device to be *longer* than the one originally emitted, this means that Andrew's is "receding" from the sheriff. So Andrew's driving away from the sheriff, thinking "#*&$%@!!!#" because he saw the sheriff measuring his speed. In the meantime, while chewing some tobacco, the sheriff pulls out his notepad and scribbles frantically:

$$z = \frac{\lambda - \lambda_0}{\lambda_0} = \frac{10.000001 - 10}{10} = \frac{10^{-6}}{10} = 10^{-7} \qquad (8)$$

so then he goes

$$\begin{aligned} z &\approx \frac{v}{c} = 10^{-7} \Rightarrow v = c \cdot 10^{-7} = 3 \cdot 10^8 \cdot 10^{-7} \text{ m/s} && (9) \\ &= 30 \text{ m/s}. && (10) \end{aligned}$$

The sheriff then stares at his notebook and doesn't start his car since he thinks the figure he got is in miles per hour. After all, Andrew remembers the sheriff saying to him once that "boyyy, in good ol' Texas we ain't doing that metric thing..."[1].

[1] For the record 30 m/s= 108 km/hr. = 67.11 mph